

Homework # 5

Math 427/527

Note : Math 427 students may do the Math 527 questions for extra credit. You may work in pairs on this assignment, but pairs can only be two 427 students or two 527 students but not mixed pairs.

If you work together (pairs of two only), *you may turn in a single homework with both names.*

All plots must have axes labels, and a title. Also, be sure to use appropriate axis limits for each plot. Make your plots interesting!

1. Is the given function even or odd or neither even or odd? Find its Fourier series. Show the details of your work.

(a) $f(x) = x^2$, $-1 < x < 1$, $p = 2$.

- (b) Using the results you found above, show that

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots = \frac{\pi^2}{6}$$

2. The following problems refer to the function $f(x) = \pi - x$, $0 \leq x \leq \pi$.

(a) Find the cosine series for $f(x)$.

(b) Find the sine series for $f(x)$.

(c) Create plots for the partial sums for each series, using 20 terms.

3. Find the steady state oscillations of $y'' + cy' + y = r(t)$ with $c > 0$ and

$$r(t) = \begin{cases} -1 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$

and $r(t + 2\pi) = r(t)$. Assume the spring constant is $k = 1$.

4. Find a trigonometric polynomial $F(x)$ of the form

$$F(x) = A_0 + \sum_{n=1}^N A_n \cos(nx) + B_n \sin(nx)$$

for the function $f(x) = x^2$, $(-\pi < x < \pi)$. Compute A_n and B_n so that the square error is a minimum. Compute the minimum errors for $N = 1, 2, 3, 4, 5$.

5. Use Parseval's identity to show

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8} \approx 1.233700550$$