

## Practice #6 - Linear Algebra

1. Answer questions about the matrix  $A$ , given as

$$A = \begin{bmatrix} 1 & -2 & 7 & 5 \\ -2 & -1 & -9 & -7 \\ 1 & 13 & -8 & -4 \end{bmatrix} \sim B = \begin{bmatrix} 1 & 0 & 5 & 19/5 \\ 0 & 1 & -1 & -3/5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Verify that the row-reduced form is correct.

(b) The null space of  $A$  is a subset of \_\_\_\_\_.

(c) Which of the following describes  $\text{null}(A)$ ?

$$(1) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s_1 \begin{bmatrix} -5 \\ 1 \\ 1 \\ 0 \end{bmatrix} + s_2 \begin{bmatrix} -19/5 \\ 3/5 \\ 0 \\ 1 \end{bmatrix}$$

$$(2) \text{span} \left\{ \begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 19/5 \\ -3/5 \\ 0 \end{bmatrix} \right\}$$

$$(3) \left\{ \begin{bmatrix} -5 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -19/5 \\ 3/5 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$(4) \text{span} \left\{ \begin{bmatrix} -5 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -19/5 \\ 3/5 \\ 0 \\ 1 \end{bmatrix} \right\}$$

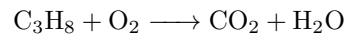
(d) What is a *basis* for  $\text{null}(A)$ ?

(e) What is the *dimension* of  $\text{null}(A)$ ? \_\_\_\_\_

(f) What is the nullity of  $A$ ? \_\_\_\_\_

2. Examples of null spaces

3. The following chemical reaction



show how propane and oxygen are converted to  $\text{CO}_2$  and water. We can balance this equation by solving  $A\mathbf{x} = \mathbf{0}$  where matrix  $A$  is given by

$$A = \begin{bmatrix} 3 & 0 & -1 & 0 \\ 8 & 0 & 0 & -2 \\ 0 & 2 & -2 & -1 \end{bmatrix} \sim B = \begin{bmatrix} 1 & 0 & 0 & -1/4 \\ 0 & 1 & 0 & -5/4 \\ 0 & 0 & 1 & -3/4 \end{bmatrix}$$

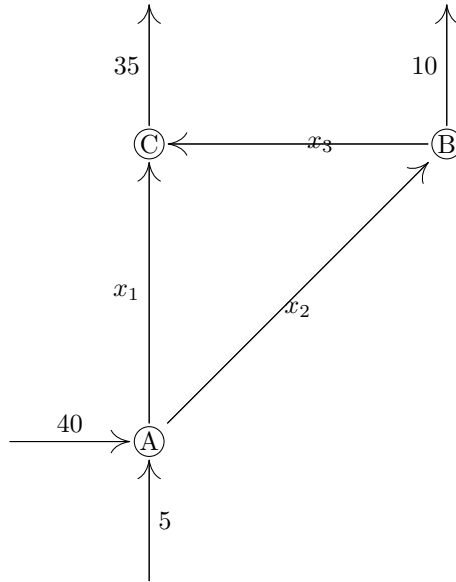
(a) Verify that the above matrix  $A$  and the row-reduced matrix  $B$  are correct.

(b) What is the null space of  $A$ ?

(c) Describe a basis for  $A$ .

(d) Interpret  $\text{null}(A)$  in terms of the chemical reaction problem.

4. **Traffic flow.** On the last exam, you solved the traffic flow problem



by solving the problem  $A\mathbf{x} = \mathbf{b}$  for

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 45 \\ 10 \\ 35 \end{bmatrix}$$

(a) Verify that the above matrix system is correct.

(b) Row-reducing the augmented matrix  $[A|\mathbf{b}]$ , one obtains the matrix  $B$

$$A \sim B = \begin{bmatrix} 1 & 0 & 1 & 35 \\ 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(c) Write down the solution in *vector form*.

(d) What is the null space of  $A$ ?

(e) What role does the null space play in this problem?