

Practice #4 - Linear Algebra

1. A subset S in \mathcal{R}^n is a *subspace* if S satisfies the following three conditions :

- (a) S contains $\mathbf{0}$, the zero vector,
- (b) If \mathbf{u} and \mathbf{v} are in S , then $\mathbf{u} + \mathbf{v}$ is also in S .
- (c) If r is a real number and \mathbf{u} is in S , then $r\mathbf{u}$ is also in S .

A subset S that satisfies (b) is said to be **closed under addition**, and if it satisfies (c), then it is **closed under scalar multiplication**.

Answer the following by checking if each of the three conditions above hold for the subset S .

- (a) Let $S = \text{span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \dots, \mathbf{u}_n\}$ be in \mathcal{R}^n . Is S a subspace? Check each of the three conditions

- (b) Is the set of all solutions $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ to the equation $2x_1 + 3x_2 = 0$ a subspace?

- (c) Is the set of all solutions $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ to the equation $2x_1 + 3x_2 = -5$ a subspace?

- (d) Does the set of all vectors of the form $\begin{bmatrix} a \\ 2 \\ 3b \end{bmatrix}$ (where a, b and c are real numbers) form a subspace?

- (e) Is the set of all solutions $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ to the equation

$$\begin{aligned} -3x_1 + 2x_2 &= 17 \\ x_1 - 5x_2 &= -1 \end{aligned}$$

a subspace?

2. Show that the set of all solutions \mathbf{x} to the homogeneous system $A\mathbf{x} = \mathbf{0}$ forms a subspace. This is a special subspace called the **null space** of A , denoted $\text{null}(A)$.

3. Solve for X ! Assume all matrices are $n \times n$ and invertible. The matrix I is an $n \times n$ identity matrix.

$$(D + BX)^{-1}AX = I$$

4. (**Markov matrices**). A medium sized town has three electric bike stations designated A, B, and C. Riders checking out bicycles can return them to any one of three stations. Rider history shows that bikes borrowed from each station are returned to a given location based on the following probabilities:

| | | Borrowed | | |
|----------|---|----------|-----|-----|
| | | A | B | C |
| Returned | A | 0.4 | 0.1 | 0.2 |
| | B | 0.3 | 0.7 | 0.7 |
| | C | 0.3 | 0.2 | 0.1 |

After a summer of bike users checking out bikes, what is the expected distribution of bikes at the three rental locations?