Practice # 2

Let $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \ldots, \mathbf{u}_m$ be a set of vectors in R^n. The **span** of this set is denoted

$$\text{span}\,\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_m\} \tag{1}$$

and is defined as the set of all possible linear combinations

$$x_1\mathbf{u}_1 + x_2\mathbf{u}_2 + x_3\mathbf{u}_3 + \ldots + x_m\mathbf{u}_m. \tag{2}$$

for any real numbers $x_1, x_2, \ldots, x_m$.

How do you determine if a set of m vectors $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_m\}$ spans $\mathcal{R}^n$?

- The vectors $\mathbf{u}_j$ must be in $\mathcal{R}^n$. That is, each vector must have n components.
- There must be at least n vectors in the set, or $m \geq n$.
- The echelon form of the matrix with columns formed from the m vectors must have *pivot in every row.*

In each of the following examples, use the checklist above to determine if the following sets span the Euclidean space of the given dimension. Provide a justification for your reasoning.

1. Does $\left\{ \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$ span R^2 ?

2. Is the following true or false? Why or why not?

$$\text{span}\left\{ \begin{bmatrix} -4 \\ 1 \end{bmatrix} \right\} = R^2 \tag{3}$$

3. Do the vectors $\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ span R^2 ?

4. Another way to describe **span** is to say that a set of vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \ldots, \mathbf{u}_m$ spans $\mathcal{R}^n$ *if and only if* the vector equation

$$x_1\mathbf{u}_1 + x_2\mathbf{u}_2 + x_3\mathbf{u}_3 + \ldots + x_m\mathbf{u}_m = \mathbf{b} \tag{4}$$

has a solution for every vector $\mathbf{b} \in \mathcal{R}^n$. Show that the vectors $\begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ span R^2 by showing that the following vector equation can be solved for any b_1, b_2.

$$x_1 \begin{bmatrix} -2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{5}$$

5. The fertilizer company has two kinds of fertilizer,

$$\mathbf{v} = \begin{bmatrix} 29 \\ 3 \\ 4 \end{bmatrix} \quad \text{and} \quad \mathbf{p} = \begin{bmatrix} 18 \\ 25 \\ 6 \end{bmatrix}, \tag{6}$$

where each component represents pounds of nitrogen, phosphoric acid and potash, respectively, per 100 lb bag. Can the company produce any possible mixture by combining bags of **v** and bags of **b**? Justify your answer using the language of linear algebra, including the term "span".

6. Do the columns of the matrix A span $\mathbf{R}^3$?

$$A = \begin{bmatrix} 1 & 3 & -1 \\ -1 & -2 & 3 \\ 0 & 2 & 5 \end{bmatrix} \tag{7}$$

7. Use WolframAlpha to determine if the claimed equality is true.

$$\text{span}\left\{ \begin{bmatrix} 4 \\ 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 7 \\ -4 \\ 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ 2 \end{bmatrix} \right\} = \mathcal{R}^4 \tag{8}$$