

Homework #3

ME 471/571

All of your work for this assignment should be turned in using Jupyter Notebooks. Use 'nbconvert' to convert your notebook to PDF.

- Make sure your plots show up in your PDF.
- Use `print(df)` to print out contents of a Pandas DataFrame.
- Provide descriptive text for each problem. Clearly identify which problem you are solving in which section. Use "anchor links" to provide a table of contents to show your work.

Please submit the version as well as the original notebook.

1. **1d Heat solver.** Convert the code `heat_serial.c` (available on the class Git repo) to a parallel code. Your code should take as input the mesh resolution and number of time steps to output.

Use the Jupyter notebook `heat.ipynb` (also on the Git repo) to plot your results. Feel free to modify this notebook, if you like.

- Choose the mesh resolution N so that it evenly divides the number of processors you choose (i.e. $N = 2^p$ is a good choice.)
 - Use "no-flux" boundary conditions.
 - You should provide convincing evidence that your serial code and parallel code produce the same results.
 - Use rank 0 and `MPI_Gatherv` to write out your output.
 - Show your timing results for $N = 2048$. What kind of speed-up do you get?
2. **2d Heat solver.** Write a serial 2d heat solver. Use this to solve the heat equation in $[-1, 1] \times [-1, 1]$. Use a Gaussian as your initial condition, given by

$$q(\mathbf{x}, t) = \exp(-10(x^2 + y^2))$$

- You may use either to use either C or Fortran for updating the solution.
3. **Spiral waves.** Reaction diffusions produce very visually appealing patterns. For this problem, we will solve a system whose solution exhibits a traveling wave solution.

The Barkeley model for spiral waves is given by the reaction-diffusion system

$$u_t = \nabla^2 u + \frac{1}{\epsilon} u(1 - u) \left(u - \frac{v + b}{a} \right)$$
$$v_t = u - v$$

where $u(x, y, t)$ and $v(x, y, t)$ are chemical concentrations in an excitable media. For this problem, you should solve the spiral wave model on a domain $[-L, L] \times [-L, L]$, for $L = 20$. Use no-flux boundary conditions on the boundary of the domain. A reasonable choice of parameters is $\epsilon = 0.02$, $a = 0.75$ and $b = 0.01$. Solve over the time span $[0, 40]$. You can initialize the problem using

$$u(x, y, 0) = H(y)$$
$$v(x, y, 0) = H(x)$$

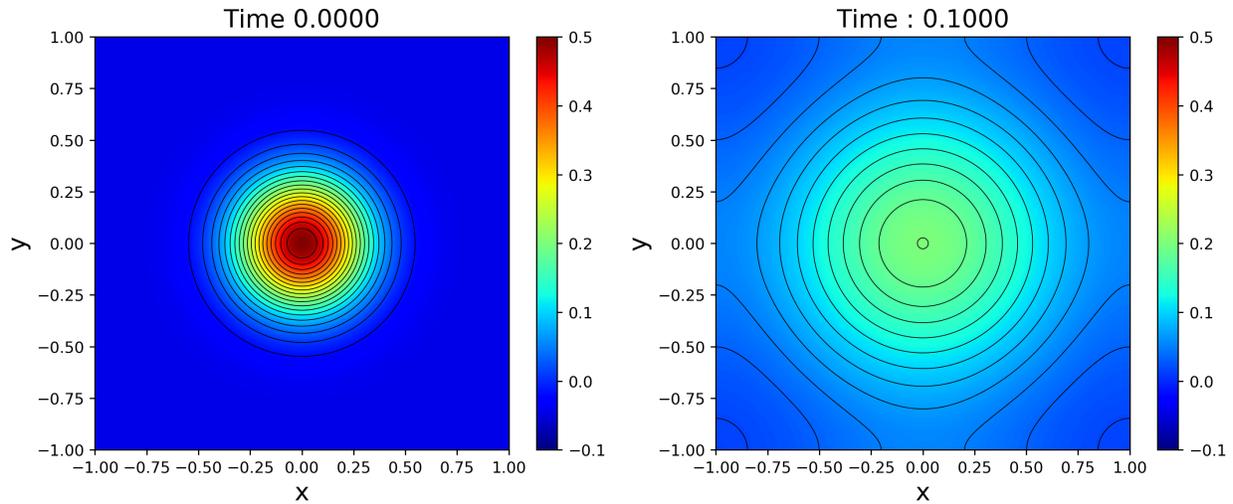


Figure 1: Solution to the heat equation at time $t = 0$ (left) and time $t = 0.1$ (right). These were produced using the notebook `heat2d.ipynb`.

where $H(x)$ is the Heaviside function defined as

$$H(s) = \begin{cases} 1 & s > 0 \\ 0 & \text{otherwise} \end{cases}$$

Use the Forward Euler Method you wrote for your 2d heat solver and write out the solutions for enough plots to see the spiral behavior. You may use the notebook `spiral.ipynb` (on the git repo) for plotting.

If you find your code blows up, you may have a stability issue with the time step. The stable time step we choose for the heat equation does not take into account any source terms, and so may not be appropriate for this problem. You can try setting the time step smaller (choose, for example a factor of 0.7 when computing the stable time step) or choose a finer grid, e.g. 256×256 .

4. **Parallel Reaction-Diffusion.** Start thinking about how to parallelize your 2d codes. You will be asked to do this for Homework #4.

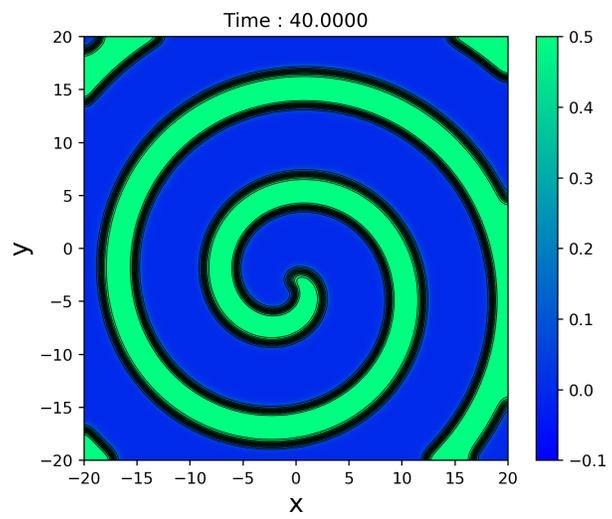


Figure 2: Spiral wave solution for excitation variable u computed in Problem 3.